Interlude 1: Exercise 4

Explanation

This exercise is a also straight algebraic manipulation, providing practice in calculating with vectors.

Hint

Recall the definition of the magnitude of a vector, the definition of the dot product, and the use of trigonometry in calculating the angle between vectors.

Answer

The magnitude of a vector can be calculated using Eq. (3).

$$\left|\vec{A}\right| = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2} = \sqrt{2^2 + (-3)^2 + 1^2} = \sqrt{4 + 9 + 1} = \sqrt{14} \approx 3.74166$$

and

$$\left|\vec{B}\right| = \sqrt{B_x{}^2 + B_y{}^2 + B_z{}^2} = \sqrt{(-4)^2 + (-3)^2 + 2^2} = \sqrt{16 + 9 + 4} = \sqrt{29} \approx 5.38516.$$

The dot product can be calculated using

$$\vec{A}\cdot\vec{B} = A_x\,B_x + A_y\,B_y + A_z\,B_z = (2)\,(-4) + (-3)\,(-3) + (1)\,(2) = -8 + 9 + 2 = 3.$$

The angle between the vectors can be calculated by using the other definition of the dot product,

$$\vec{A}\cdot\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta,$$

we can solve this for $\cos\theta$,

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{\left|\vec{A}\right|\left|\vec{B}\right|} = \frac{3}{(3.74166)\,(5.38516)} \approx 0.148888$$

then we take the arccos θ, to find that $\theta \approx 1.42135$ radians, or about $81.4374°$.